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Stephen Cole Kleene – a reminiscence

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Stephen Kleene and I were both born in 1909 and both graduated from college in 1930. We first met in the fall of 1933, when I came from my Postdoc at Yale to visit Princeton. There I also met Alonzo Church, Kurt Gödel, Barkley Rosser and Steve. It was an exciting and heady time for the development of mathematical logic. Steve and I, with common interests in logic and a common New England background, at once hit it off. We remained close friends for the rest of his life.

Steve had come to Princeton after graduating from Amherst College, summa cum laude, in 1930. This was the right time and the ideal place to study mathematical logic. Before that time there had been essentially no logicians in American departments of mathematics. However, at Princeton, Oswald Veblen (in his retiring address as President of the AMS) had stated clearly that it was essential that mathematicians should take the lead in the development of logic. Two of Veblen's Ph.D. students (A.A. Bennett and Alonzo Church) had written theses on logic; Church had stayed on as a member of the Princeton faculty. There he had invented the lambda calculus as a possible new foundation for mathematics, and there his students Kleene and Rosser were enormously active. And at the time Gödel's theorems opened new prospects for logic. Steve proceeded to develop some of these prospects.

Kleene left Princeton in 1935 to become an instructor at the University of Wisconsin in Madison. We met again there, in 1937, when we first engaged in a little mountain climbing, an occupation at which he was better than I. We met also at AMS meetings, especially on one notable occasion in New York, where we boycotted an official banquet to go to a steak house which served enough steak to match Steve's appetite. Then when Dorothy and I lived at 7 Avon Street in Cambridge, Steve came often to visit. An ordinary single bed was not long enough to match his height, so we bought a double bed for a guest room – so that he could sleep aslant on his visits.

Then about 1940 there was to be an AMS meeting at Dartmouth College. Steve and I decided not to miss this opportunity to examine the Presidential range and its reputation for bad storms. Before the meeting we met in North Conway and climbed to the top of Mount Washington, luckily in good weather. After staying overnight in

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a hut we picked up another young climber and went on to ascend the other peaks of the range. Before we could reach the last peak, Mount Madison, the weather turned uncertain, but we all three persisted in this last ascent. On top of Madison, as I was at the upper edge of a slanting rock, I suddenly found myself slipping without control and then heard Steve shout "Get down! It's lightning!". Steve was already running down the slope; as I started to follow him, I looked for our third man – to find him lying unconscious, and twitching. I at once began to give him artificial respiration. Steve quickly came back, to ask "Is he breathing?". He was, but only slowly. Then Steve, who had the longer legs, went down again to the Madison Notch hut for help, while I stayed, and the victim finally became conscious again. He was much mystified by his new mittens (mine). Which Steve came back with help, we got the victim down to the hut. It turned out that the lightning had taken out a small piece of his scalp and had left a big bruise on his left heel. We dried our clothes before the fire and then slept off the experience.

By the next morning, our companion was partially recovered. We took him down to his car and then to the hospital near the elegant Eastern Slopes Inn, where he was the Bell Captain. I had to walk across the magnificant lobby to tell this tale to the manager. He guessed it in advance; my pace in walking was constrained because drying my trousers had also burned a big hole in the seat!

Later our Bell Captain recovered and wrote us in thanks. Steve and I agreed that climbing mountains is fine, but not in a thunderstorm.

By 1941, Steve was still an Assistant Professor at Wisconsin, so was tempted by the offer of an Associate Professorship at Amherst, his Alma Mater. The then dean at Wisconsin assured him that he would eventually return, perhaps even as dean.

In 1942, Steve married Nancy Elliott; we were happy to have them visit together when Dorothy and I were temporarily in New York for war research. About that time, Steve joined the U.S. Navy as Lieutenant (J.G.); he rose to be Lieutenant Commander by the time of his discharge in 1946 – when he returned to Madison as Associate Professor of Mathematics.

Earlier, when I was the editor for the Carus Mathematical Monographs, I was keenly aware that there was not an adequate up to date text on mathematical logic. Hence I proposed to Kleene and Rosser that they should write such a text. In Kleene's case this led to the preparation of his magnificant volume "Introduction to Metamathematics" (1952). This book provided the decisive formulation of the properties of recursive functions, a firm and clear presentation of Gödel's theorem and many other important results. I have often returned to consult this book; I still take pleasure in Steve's acknowledgement in the preface of my early encouragement of the project.

At the University of Wisconsin, Kleene trained many students and encouraged collegues. He did become Dean of the College of Arts and Sciences (1969–1974). When I visited him in his then office, I found him still full of interest in mathematical logic. He was also the designer of the new Van Vleck building for the mathematics department – and at once juncture he guided me through the building when it was not yet ready – but when he had full access. Then I noted on the top floor the magnificent

common room with its view of the lake. Later other friends there have observed that all the individual offices were quite small – Steve had planned them so, just to be sure that there would never come a time when two professors were assigned to one office!

It was clear to many of us that much of Steve's hard intellectual clarity had one source in his family background in the Kleene farm in Union, Maine. By some sad omission, I never did get to climb Mount Katadin with him as guide. But I did visit the ancestral Kleene farm. One summer, just as I was landing from a cruise on Penobscot Bay, I ran across Steve, standing on the dock in Camden. He invited me to come up to the farm; with Nancy we drove over to Union. Then I saw the venerable farm-house, inspected the out buildings and walked down to the nearby lake. So I came to better understand Steve's attachment to the family tradition rooted there.

In more recent years, I saw Steve not on the mountains, but in Washington, where he enjoyed attending the meetings of the National Academy of Sciences with his wife Nancy, and then, after her death, with his second wife Jeanne. I also visited him in Madison, where he continued to be proud of his house on Wood Lane – and of the fence which he had built there to please Jeanne. My daughter Gretchen and I both recall with delight his dry sense of humor.

I return to Kleene's beginnings in logic. I have no direct information about Steve Kleene's undergraduate background in mathematics and logic. But let me recall the general situation then.

The ideal of logical rigor was well accepted in most mathematical circles. After the foundation of the calculus by Cauchy and Riemann, Weierstrass had popularized the systematic use of epsilons and deltas in the definition of continuity. This amounted in fact (but not in the general recognition) to an understanding of the importance of quantifiers. This technique was then accessible to undergraduates; for example I learned it in 1927 from the careful text in advanced calculus by Edwin B. Wilson. Other common uses of meticulous arguments appeared in rigorous courses on complex variables and in research on point-set topology. I do not know where Kleene first learned such ideas, but it is clear that rigor came readily to him – as one may see in the remarkable care and precision displayed in his "Introduction to Metamathematics" – as in the definition there of recursive functions and in his clear presentation of Gödel's incompleteness theorem.

Mathematicians in 1930 also had a firm background in axiomatics, as represented by Hilbert's axioms for Euclidean geometry and those of Veblen for projective geometry. There were at hand axioms for the real numbers and for Boolean algebra, but these matters did not seem to be of central interest. It was known that Zermelo had axiomatized set theory, but that was somehow subordinate to Zermelo's use of the axiom of choice to prove that every set can be well-ordered. Skolem, Fraenkel and von Neumann had polished up the axioms for the theory, but they were still a mathematical side-line. There were various competing doctrines about the foundation of mathematics. Intuitionism had lost ground, perhaps because Brouwer's ideas were hard to understand – though Kleene was later able to illuminate this from

the viewpoint of recursive functions. Hermann Weyl had written "Das Kontinuum" but did not really follow it up; at any rate he did not press such views in the course which he gave (and which I wrote up) in Göttingen in 1932. David Hilbert, with the assistance of Paul Bernays, was heroically defending all of classical mathematics in terms of formalism.

Logic provided one very visible and apparently solid foundation – all three volumes of it – in Whitehead and Russell's *Principia Mathematica*. There it was, laid out in complete detail with only a smidgeon of informality when it came to distinguishing between axioms and rules of proof. But there it was, a monument. My professor of logic at Yale, 1928 (F.S.C. Northrop, a student of A.N. Whitehead), had assured me that the logic in Principia settled it all. I settled for the first volume; innocent of the remaining two, I became convinced that logicism did provide the ultimate foundations for mathematics. W.V. Quine, studying at Oberlin at the same time, was evidently similarly impressed. Thus logicism in 1930 appeared to be the promising direction for mathematical logic. Left to itself, it would have become a dull direction.

But that was not to be so. Steve Kleene, as well as Barkley Rosser, Alan Turing and others entered the field of mathematical logic with the advent of Gödel's incompleteness theorem. That theorem squarely addressed the central problem of foundations. In Germany, it led to proof theory, especially at the hands of my fellow students in Göttingen, Gerhard Gentzen and Kurt Schütte. In Princeton, it led to the basic ideas about recursive functions, which Kleene developed and applied so brilliantly. Kleene's work came at a decisive turning point for the study of mathematical logic. Like his beloved mountains, his work was hard, clean and definitive.

P.S. Paul Kleene, one of Steve's sons, has lent me some of Steve's early letters to his mother (then living in Hartford, CT). In 1930, about to graduate from Amherst, he asked her advice about accepting an offer from Princeton: a part-time instructorship with a stipend of $1000 (plus tuition), requiring six hours a week of teaching and one evening conference. He accepted; his graduate courses that year were real variable and algebra. One of his subsequent letters explains to his mother the axioms for an integral domain.

He spent part of the winter of 1933–34 at the farm in Maine – and asked his mother to send him a book; Kephart's "Camping and Woodcraft".

There was mathematical excitement. Early in the morning of November 7, 1934, he wrote his mother:

"Rosser and my article is deadly. It is only 8 pages typed and prob. same printed, but it would require reading a couple of hundred pages perhaps to make full check up on it all. One sentence takes 10 pages to prove".

Then on November 14, 1934, he wrote again:

"We, Rosser and I, just turned in our paper again. One day after submitting it before, Curry came out with an article in which he added some more axioms to his system. However, our paper had said that if he added a certain three axioms, his

system would become inconsistent. Those very three axioms were among those in the new paper. So we recalled the paper and changed it to say Curry's system *is* inconsistent. Doubtless the shock will be rather severe to Curry. This is the paper in which it is also proved that Church's system is inconsistent. The one sentence in which we said something that required 10 pages proof was merely the assertion that something could be proved".

This letter clearly refers to the famous paper by Kleene and Rosser: The Inconsistency of Certain Formal Logics, *Ann. of Math. Ser.* **2**, Vol. 36 (1935) pp. 630–636.